

Principle of Reciprocity Solves the Most Important Problems in Bioimpedance and in General in Bioelectromagnetism

Jaakko Malmivuo

Tampere University of Technology, Tampere, Finland

jaakko.malmivuo@tut.fi

Abstract. Though the principle of reciprocity was invented by Hermann von Helmholtz already over 150 years ago, and though it is a very powerful tool in solving various important problems in bioelectromagnetism, it is not generally used. In impedance tomography the measurement sensitivity distribution has generally been misunderstood. This can be easily demonstrated with the principle of reciprocity. Some other applications of the principle of reciprocity are also discussed.

1. Principle of reciprocity

The principle of reciprocity is a general principle, which holds for all linear systems. It was first described by Hermann von Helmholtz over 150 years ago [1]. In brief, it may be described by noting, that the source and the measurement sites may be swapped without a change in the detected signal. For us, who work with bioelectromagnetic problems it was a great honor, that Helmholtz demonstrated this principle in connection to measuring a bioelectric source.

Helmholtz described the principle of reciprocity in the following way:

A galvanometer is connected to the surface of the body. Now every single element of a biological electromotive surface produces such a current in the galvanometer circuit as would flow through that element itself if its electromotive force were impressed on the galvanometer wire. If one adds the effects of all the electromotive surface elements, the effect of each of which are found in the manner described, he will have the value of the total current through the galvanometer.

This statement also includes the principle of superposition for the first time.

2. Sensitivity distribution in impedance tomography

2.1. Classical illustrations of measurement

In the beginning of impedance tomography analysis it was erroneously understood, that in the impedance measurement the voltage measurement with an electrode pair reflects the impedance of the area (or volume in three dimensions) bounded by the equipotential lines (or surfaces) generated by the current injection and passing through the voltage measurement electrodes. This concept is referred in several standard references even so late as 2005 [2, 3, 4, 5]

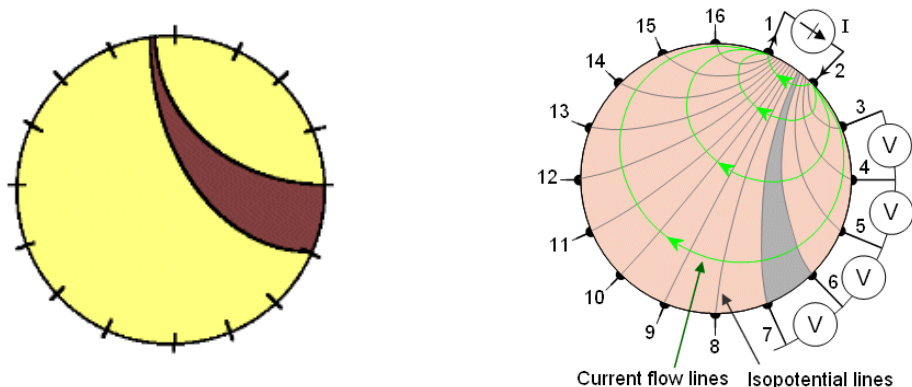


Figure 1. The logo of the Electrical Impedance Tomography (EIT) community (left). Classical but erroneous illustration of the distribution of measurement sensitivity in electrical impedance tomography (right).

2.2. Reciprocal approach

The principle of reciprocity clearly indicates that the classical concept of the region, from where the impedance is measured, can not be correct. According to reciprocity, the impedance measurement result should not change if the current feeding and voltage measurement are swapped. Therefore, the measurement region should be similar close to both electrode pairs. Furthermore, the measurement sensitivity may not be limited to an accurately bounded region, but the signal must be affected with some amount by all regions of the volume conductor.

We noted this and recalculated the measurement sensitivity distribution for neighboring, opposite, cross and adaptive methods [6]. The data is also presented as animations. This data clearly demonstrates both the symmetry and wide distribution of the impedance measurement sensitivity.

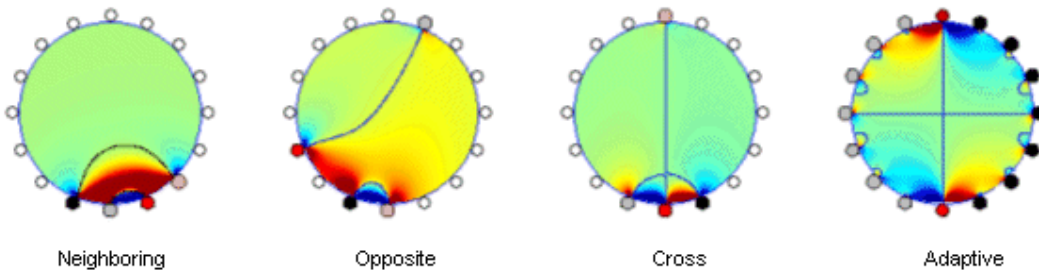


Figure 2. Distribution of the measurement sensitivity in one electrode configuration in the neighboring, opposite, cross and adaptive methods.

2.3. Positive and negative sensitivity

Figure 2 indicates the measurement sensitivity with a color code, where red is positive and blue is negative sensitivity. The concept of negative sensitivity complicates the analysis of the impedance signal. In positive sensitivity the impedance signal increases when the impedance of that region increases. In negative sensitivity the signal decreases with increasing impedance. The background for this phenomenon is, that when the lead fields (or their components) of the current and voltage leads are in the same direction the sensitivity is positive, when they are in opposite direction, the sensitivity is negative [4]. This phenomenon of positive and negative sensitivity is seen for instance in the measurement sensitivity analysis of different impedance cardiography leads, Figure 3 [7].

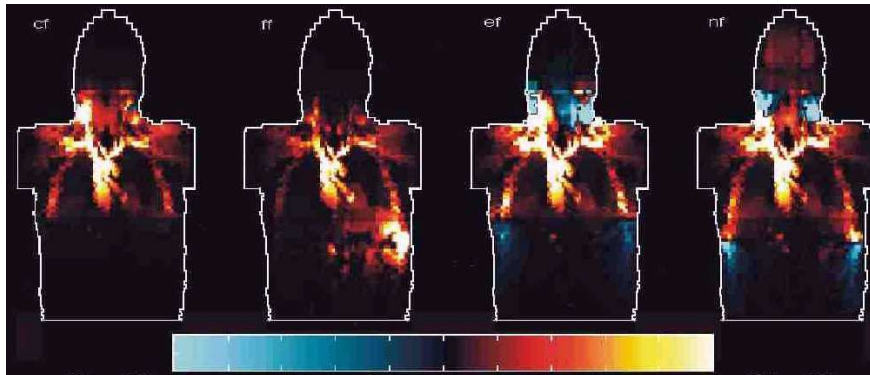


Figure 3. Distribution of measurement sensitivity in different impedance cardiography lead configurations. Red color indicates positive and blue color negative sensitivity. [7]

Note that the positive and negative sensitivity phenomenon is not unique in bioelectromagnetic problems. Similarly, if in measuring a bioelectric source the source is oriented in the direction of the detector's lead field, the sensitivity is positive. If the source is oriented in the opposite direction, the sensitivity is negative.

2.4. Resolution in impedance tomography

That the resolution of impedance tomography is so poor originates from the aforementioned distribution of the measurement sensitivity. In computer tomography with X-rays the resolution is excellent, because the detected amplitude of the signal depends on the density of the material on the narrow straight line of X-ray beam between the X-ray source and the detector. The accuracy of the back-projection is determined by this factor. If the classical understanding of the measurement sensitivity distribution had been correct, the accuracy of impedance tomography could have been improved close to the level of X-ray tomography by increasing the number of electrodes so that their distance would be extremely small.

3. Examples of other applications of the principle of reciprocity in bioelectromagnetism

The principle of reciprocity helps in solving several problems in bioelectromagnetism [8]. It also gives a very illustrative tool to understand the behavior of measurement sensitivity or stimulus intensity distribution problems. In the following some examples are given.

3.1. Distribution of measurement sensitivity and stimulus intensity

Feeding a unit current to the measurement electrodes in electric measurement or coil in magnetic measurement raises a current distribution in the volume conductor. This current, called the lead field, is exactly the same as the distribution of measurement sensitivity.

Because the lead field is a physical current field it is actually the distribution of stimulus current. Therefore the distributions of the measurement sensitivity and the stimulus current for a pair of electrodes or a coil are identical.

3.2. Reciprocally reciprocal method

The principle of reciprocity holds also for the reciprocal situation. As an application in finding the lead field for a magnetic measurement, the reciprocal current may be fed to the source dipole and signal detected with a superconducting SQUID magnetometer. Despite of this the measurement results may be considered as the reciprocal current had been fed to the coil and the signal detected with the electrodes in the volume conductor. The former way of thinking is made to understand the lead fields of the magnetic detector and the latter physical procedure is made because the SQUID magnetometer is more sensitive detector than dipolar electrodes in the volume conductor model [9].

3.3. Independence/interdependence of electric and magnetic measurements

The fundamental problem in biomagnetism is: do the magnetic measurements include information which is independent from the electric measurements?

On the basis of the Helmholtz theorem it was first believed, that the electric and magnetic measurements are independent [10]. If this were the case, with the magnetic measurement it would be possible to obtain as much new independent information as is obtained with electric ones. The practical measurements have, however, shown that this is not the case.

With the lead field theory it has been possible to demonstrate, that the electric and magnetic lead fields are independent, but the measured signals are only partially independent [4].

3.4. Reducing calculation in multichannel problems

For example in cortical potential imaging the cortical surface may consist up to 100,000 nodes and the number of electrodes may be 250. Applying the principle of reciprocity the number of calculations in cortical potential imaging decreases the number of calculations from 100,000 to 250. This means that the calculation time is decreased to 1/400.

4. Conclusion

The principle of reciprocity is not generally applied in bioelectromagnetic problems. Though the direct method is straightforward and may be applied in several problems, in many important problems the reciprocal method gives faster and more illustrative solution. In some problems it is the only possible method for obtaining the solution.

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